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APPLICATION OF SECOND ORDER REACTION

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"Application of Second Order Reaction"

Covering 12 month period from
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STATEMENT OF PROGRESS

A. Theses and Publications

1. Theses - This report.

The following theses have been approved and are currently in the final stages of preparation.

a. "Oxygen Absorption into a Liquid Flowing Over a Slime Covered Sphere," by Walter H. Scheible, Thesis, Master of Science, Cornell University, Ithaca, N.Y. An abstract of the thesis is included with this report as Appendix A.

b. "Mass Transfer and Growth Kinetics on a Slime Layer, A Simulation of the Trickling Filter," by Walter J. Maier, Thesis, Doctor of Philosophy, Cornell University, Ithaca, N.Y.

The following thesis is tentative. You will be notified of the final title and the date of publication.

c. "Biological Reaction Kinetics in Piston Flow and Completely Mixed-Continuous Flow Systems," by Parviz Monadjemi (Ph.D. Thesis).

2. Publications - This report.

A paper entitled "Developments in Biological Filtration" by V. C. Behn and P. Monadjemi was given before the Special Lectures on Advances in Water Quality Improvement, University of Texas, April, 1966. According to Dr. E. [illegible]loyna of the University of Texas the collected papers will be published by the University of Texas. Ten copies of the printed publication will be forwarded to the Division of Water Supply and Pollution Control as soon as they are received. A copy of the paper is attached to this report as Appendix C.

Our intention is to publish the work of Walter Maier as one or possibly two papers. This is under consideration at the present time. Ultimately we would like to publish the work of Parviz Monadjemi. We will advise you concerning these papers as soon as they are prepared.

3. Theses and Publications - Previous reports.

a. Discussion of "Trickling Filter Design and Performance," by V. C. Behn, W. F. Ames and K. Keshavan, Jour. San. Div., Proc. Amer. Soc. Civil Eng., 88, SA1, 31-37, January, 1962.

b. "Transient Operation of the Trickling Filter," by W. F. Ames, V. C. Behn and W. Z. Collings, Jour. San. Eng. Div., Proc. Amer. Soc. Civil Eng., 88, SA3, 21-38, May, 1962.

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c. "Kinetics of Biological Removal of Organic Wastes by Aerobic Microorganisms," by K. Keshavan, Ph.D. Thesis, Cornell-University, Ithaca, N.Y., June, 1963.

d. "Kinetics of Aerobic Removal of Organic Wastes," by K. Keshavan, V. C. Behn and W. F. Ames, Jour. San. Eng. Div., Proc. Amer. Soc. Civil Eng., 90, SA1, 99-126, Feb. 1964.

e. "Analysis of BOD Reduction in Trickling Filters," by M.A. Amado, M.S. Thesis, Cornell University, Ithaca, N.Y., Feb. 1964.

B. Results Obtained

We were able to demonstrate that the activated sludge process could be analyzed on the basis of a second order reaction. In other words, the rate at which organic waste is oxidized is a function of the waste organic matter present and the biological cell material (Thesis by Keshavan and paper A.3.d.).

We then attempted to utilize the same type of approach in working out a formulation for trickling filters (Thesis by Amado and paper A.3.b.). This analysis, which was similar to that used in the chemical engineering profession, yielded a final result which was similar to that developed by other workers who start their analysis with a simple first order relationship. Further analysis (Appendix C) demonstrated that a first order relationship would hold if trickling filters were thought of as plug flow devices while the activated sludge treatment device could be thought of as a completely mixed-continuous flow mechanism.

The thesis by Scheible (Appendix A) and particularly the thesis by Maier (Appendix B) yielded data which was very helpful in reaching a final decision as to the type of mechanism which drives a trickling filter. Maier found, using a flat screen and glucose as the carbon source, that the reaction was mass transfer (rather than reaction) controlled. Under these circumstances it should be true that conventional trickling filters equations would not be dependent upon reaction kinetics at all. In other words, the first order type equation (see Appendix C) may be used but then the K values in the equation have as their source the mass transfer rate rather than the biochemical reaction rate.

Also Maier found that the reaction rates observed on his flat plate were substantially less than the reaction rate in a batch reactor. This work tends to support the Ingram Trickling Filter as built at Greece, N.Y. Air inserted underneath a bed of trickling filter spheres as found in this unit will doubtless disturb the flow down over the spheres and this introduces agitation into the motion of the liquid. Also since the bed

is divided into six individual parts there is ample opportunity for the flow to become disrupted as each new layer of spheres is encountered. If the filter is mass transfer controlled, these two factors should definitely enhance the operation of this particular trickling filter.

In general then we have accomplished what we originally set out to do. There is still the question of application of the second order equation to what may be called an accelerated trickling filter. Mr. Monadjemi's work will doubtless shed light on this.

APPENDIX A

Abstract of Thesis

"Oxygen Absorption into a Liquid Flowing Over a Slime Coated Sphere"

by Walter H. Scheible

ABSTRACT

"Oxygen Absorption into a Liquid Flowing Over a Slime Coated Sphere"

by Walter H. Scheible

The amount of oxygen absorption into a synthetic sewage media flowing over an active slime was determined and compared with the lesser oxygen absorption when there was no simultaneous demand by an active slime. The area of absorption was that offered by a single two and one-half inch sphere. The reduction of biochemical oxygen demand effected by the slime on the synthetic sewage media was determined as relatively independent of the dissolved oxygen concentration in the flowing liquid.

A secondary undertaking involved the determination of the diffusion coefficient of oxygen into both distilled water and the synthetic sewage media. With the relative values of the diffusion coefficient, it was shown that organics in solution do have an inhibitary effect on oxygen transfer.

APPENDIX B

Abstract of Thesis

"Mass Transfer and Growth Kinetics on a Slime Layer,
A Simulation of the Trickling Filter"

by Walter J. Maier

ABSTRACT

"Mass Transfer and Growth Kinetics on a Slime Layer,
A Simulation of the Trickling Filter"

by Walter J. Maier

A simple physical model, in the form of a flat surface covered with biologically active slime, has been used to simulate and study the purification of sewage in trickling filters. A theoretical description of this model has been presented. The theory considers mass transfer and the growth rate of microorganisms as the underlying phenomena in the purification process.

It is shown that the liquid film is in laminar flow as it moves over the slime surface. Consequently, mass transfer to the slime is by molecular diffusion. Studies with glucose and minerals show that only the top surface of the slime is active in the purification process. Increasing the thickness of the slime layer above 0.048 cm had essentially no effect on glucose removal.

Process variable studies, e.g. feed rate, concentration of glucose, and temperature, show that mass transfer is rate limiting except at high feed concentrations (above 100 mg/l). This is most strikingly shown by the two-fold increase in glucose utilization observed when the liquid film is agitated to enhance mass transfer by eddy motion.

Results from the process variable study on the flat surface together with the measured growth rates of agitated batch reactors indicate that the maximum growth rate of the microorganisms is substantially greater than the rates observed on the flat surface where diffusion limits availability of nutrients. This could be a fruitful area for further research.

APPENDIX C

"Developments in Biological Filtration"

by V. C. Behn and P. Monadjemi

DEVELOPMENTS IN BIOLOGICAL FILTRATION

by

Vaughn C. Behn and Parviz Monadjemi

Special Lectures on Advances
in Water Quality Improvement

April 4-7, 1966

University of Texas
Austin, Texas

INTRODUCTION

A biological or trickling filter is a basic waste treatment process used for reducing the organic load present in municipal and industrial waste water. It consists of a bed of stones or other natural or synthetic media upon which the waste water is applied, with the subsequent growth of microbial slimes in the filter.

During the past few decades a number of empirical and mathematical models have been developed for the action of trickling filter.

In an earlier paper (Behn, 1963) an attempt was made to summarize and relate the then existing formulations. Attention was given to the work by Phelps, Velz, Schulze, Howland and Stack. In particular, the relationship between the equilibrium load contained in a saturated filter as proposed by Phelps and Velz, and the applied load which would saturate a unit depth as suggested by Stack, was demonstrated. The availability factor utilized in the National Research Council formula was reviewed from the standpoint of its physical interpretation and its role in the Stack equation for a recirculating filter.

Since then, a number of new formulations have appeared in the literature which will be reviewed in the first section of this paper along with certain of the older mathematical formulations.

The second part consists mainly of some new concepts concerning the factors which affect the performance of a biological system such as trickling filter. The effect of flow patter on the efficiency of a system undergoing a first order reaction is described. Attention is called to the importance of concentration of bacteria in most biological systems no matter what the patter of flow is. Certain observations are made on the relationship of flow to the concentration of bacteria in determining the efficiency of a trickling filter system.

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VARIOUS MATHEMATICAL FORMULATIONS

Single Stage Filter

Many workers have examined the mechanism of BOD removal in trickling filter.

Velz (1948) proposed a "basic law" for the performance of trickling filter. He postulated that the amount of BOD removed per unit depth was proportional to the BOD remaining.

$$\frac{dL}{dD} = K_D L \tag{1}$$

or, upon integration

$$\frac{L_e}{L_o} = e^{-K_D D_o} \tag{2}$$

where L_e is the removable BOD of the effluent, L_o is the removable BOD in the influent, D_o is the depth of the filter, and K_D is a rate constant.

Velz and later Stack (1957) assumed that there is a given loading above which the purification rate, $\frac{dL}{dD}$, remains constant. Phelps and Velz (1948) stated that the limiting BOD load is a function of storage capacity of the bed. Assuming that L_f, the BOD load stored in the bed, is oxidized according to a first order reaction, and that new BOD is being added at the rate L_a in the filter of depth D_o

$$\frac{dL_f}{dD} = -K_f L_f + L_a (1 - e^{-K_D D_o}) \tag{3}$$

At limiting BOD load $\frac{dL_f}{dD} = 0$ and maximum load to be applied is

$$(L_a)_{max} = \frac{K_f L_f}{(1 - e^{-K_D D_0})} \tag{4}$$

Stack (1957) introduces a more refined concept, that of filter saturation at various depths in sequence.

Fair and Geyer (1956) presented the rate of BOD removal in a filter as

$$\frac{dL}{dD} = -K_D \left(\frac{L}{L_o}\right)^n L \tag{5}$$

or alternatively

$$\frac{dL}{dt} = -K_t \left(\frac{L}{L_o}\right)^n L \tag{6}$$

where L_o is the initial removable BOD applied to the filter, and n is a coefficient which is a measure of the non-uniformity of the rate of purification. When $n = 0$, equation (5) reduces to Velz's law, i.e., first order reaction.

Both Stack and Velz worked with depth as the parameter. W. E. Howland (1957) proposed that the rate of BOD removal was a function of the time of contact, t, i.e.,

$$\frac{dL}{dt} = -K_t L \tag{7}$$

which integrates to

$$\frac{L_e}{L_o} = e^{-K_t t} \tag{8}$$

The contact time, t, in a column cubically packed with spheres is (Sinkoff et. al, 1959)

$$t = 1.3 \left(\frac{3\nu}{g}\right)^{\frac{1}{3}} \left(\frac{A_v}{Q}\right)^{\frac{2}{3}} D_o \qquad (9)$$

where

ν = kinematic viscosity

g = acceleration due to gravity

A_v = specific surface area

Q = volumetric rate of flow per unit area

Sinkoff, Porges and McDermott (1959), making use of experimental results, concluded that the contact time in general is given by

$$t = C_1 \left(\frac{A_v}{Q}\right)^{n'} D_o \qquad (10)$$

where C_1 is a constant and the exponent n' varies with the type of filter medium and the hydraulic characteristics of the system. For example, Sinkoff, Porges and McDermott (1959) found n' to range between 0.53 for porcelain spheres to 0.83 for glass spheres.

Studies by Schulze (1960) on a screen filter gave a value of n' equal to 0.66. Schulze combined equations (10) and (8) to obtain

$$\frac{L_e}{L_o} = e^{-K' \frac{D_o}{Q^{.66}}} \qquad (11)$$

in which the effect of specific surface area is apparently assumed to be constant and is included in the value of K'.

Eckenfelder (1961) modified equation (11) and took into consideration that C_2, the amount of active surface film covering the filter medium, decreases with depth, i.e.,

$$C_2 \propto \frac{1}{D^m} \tag{12}$$

By making proper substitution, Eckenfelder obtained the following:

$$\frac{L_e}{L_o} = e^{-K''} \frac{D_o^{1-m}}{Q^{n'}} \tag{13}$$

However, the constituents of domestic wastes are not removed at the same rate. The components which are more readily oxidizable are utilized with a faster rate. Because of this fact, Eckenfelder modified equation (13) to a retardent form in order to describe the over-all reduction of BOD. His final equation is as follows:

$$\frac{L_e}{L_o} = \frac{1}{1 + \frac{C_3 D^{1-m}}{Q^{n'}}} \tag{14}$$

Eckenfelder determined the following values for the constants of this equation

$$\frac{L_e}{L_o} = \frac{1}{1 + \frac{2.5\ D^{.67}}{Q^{.5}}} \tag{15}$$

where the depth D is feet and the hydraulic rate Q is in million gallons per acre per day.

Equation (14) is apparently obtained from equation (13) by expanding the latter in the form of $\frac{1}{e^x}$ and ignoring all

but the first two terms in the denominator which gives it the retardent form.

Ames, Behn and Collings (1962) developed a mathematical model for the mechanism of trickling filter. The development of this model was based on certain hypotheses resulting from theoretical considerations of filter operation, that were similar in character to those that occur in chemical engineering treatment of packed columns. They described a component of the waste liquor, the BOD, being adsorbed into the biological slime, and then undergoing a first order biochemical reaction.

Through mass balance techniques, partial differential equations were set up and solved for the transfer of BOD. The resulting steady state equation relating the fraction of BOD in the bulk stream at any depth D is

$$L = \beta + (L_o - \beta) \exp \frac{-R}{(R + 1)} \frac{kA_v}{\rho Q} D \qquad (16)$$

where

$R = \frac{K}{\alpha k A_v}$ and

K = first order reaction constant

k = mass transfer coefficient

A_v = specific surface area

α = equilibrium constant

β = nontransmissible fraction of BOD

ρ = liquid density

Amado (1964) wrote out the complete expression for the exponent as

$$\frac{-R}{R+1}\,\frac{kA_v}{\rho} = \frac{-\frac{1}{\rho}}{\frac{\alpha}{K}+\frac{1}{kA_v}} \qquad (17)$$

This last form shows the constants in the exponent to be in the familiar form of the over-all mass transfer coefficient which is here called K_m.

$$\frac{1}{Km} = \frac{1}{kA_v} + \frac{\alpha}{K} \qquad (18)$$

The over-all mass transfer coefficient consists of two individual resistances, one to adsorption, $\frac{1}{kA_v}$, and the other to reaction, $\frac{\alpha}{K}$. In a manner similar to Eckenfelder's mean active film concept, the over-all mass transfer coefficient was assumed to be inversely proportional to some power of depth and also by analogy to gas adsorption processes it was assumed to be proportional to the volumetric flow rate per unit area raised to some power. Using these two assumptions, the following final equation was obtained:

$$L = \beta + (L_o - \beta)\exp -K_1\frac{D^{m'}}{Q^{n'}} \qquad (19)$$

where K_1, n', m' are constant for a specific waste.

The constants in equation (19) were determined by Amado, through experimental studies run on an eight foot high laboratory trickling filter model, to be

$$\frac{L}{L_o} = 0.1 + 0.9\exp\frac{-0.61D^{0.628}}{Q^{0.440}} \qquad (20)$$

where D is in feet and Q is in cubic feet per square foot per hour. This equation checks closely with the one proposed by Eckenfelder.

Atkinson, Busch and Dawkins (1963) considered the trickling filter as a "complex system of mass transport and reaction kinetics." They based their analysis on a steady state filter. This equilibrium condition is attained after a certain period of time which is dependent upon the applied organic load, the hydraulic loading and the "dimensions of the flow path." Furthermore, for the sake of simplicity, they assumed a first order process. Through a mass balance about an element of volume dV for a single stage filter, the following expression resulted

$$\frac{dL}{dD} = -KLQhW \tag{21}$$

where

h = film thickness

W = film width

which integrates to

$$L = L_o \, e^{-K \frac{hWD}{Q}} \tag{22}$$

if $\frac{hWD}{Q}$ is considered as an equivalent detention time t, the above expression is the same as formula (8).

More recently, Galler and Gotaas (1964) proposed a mathematical model for trickling filter using a large number of data and multiple regression analysis. They included recirculation, temperature, hydraulic rate, depth and organic

loading as the important factors whose effects were to be determined. They obtained the following formula

$$L'_e = \frac{0.31\ L_o^{1.19}\ (1 + N)^{0.28}}{(1 + D_o)^{0.67}\ T^{1.15}\ [(1 + N)Q_2]^{0.06}} \qquad (23)$$

L'_e = BOD load of effluent pounds per acre per day

L'_o = BOD load of influent pounds per acre per day

T = temperature in degrees Centigrade

N = ratio of recirculation to inflow rate

Q_2 = million gallons per acre per day, undiluted

Note that the exponent of the flow into the filter is close to zero which means that the hydraulic rate, as shown by equation (23), is not an important factor, a result which is at variance with the finding of others.

Recirculating Filters

The first attempt to quantitatively define the effect of recirculation was in the NRC formula (1946). The average number of passes through the filter, F', was given as

$$F' = 1 + \frac{R}{I} \qquad (24)$$

where R is the rate of recirculation and I is the rate of incoming flow.

However, it is a fact that on each pass, the difficulty with which the organic matter is removed will increase. Correspondingly, a removability factor f is introduced and

the average number of "effective" passes, F_1 is obtained as follows

$$F_1 = \frac{1 + \frac{R}{I}}{[1 + (1 - f)\frac{R}{I}]^2} \tag{25}$$

The average number of "effective" passes is maximized when

$$\frac{R}{I} = \frac{2f - 1}{1 - f} \tag{26}$$

The equations (24) and (25) do not relate recirculation with reaction kinetics. Velz (1948) did this assuming that in equation (2) the value of D_o would be increased directly by the right-hand side of equation (24). For more discussion and other derivations, see the 1963 reference by Behn.

With recirculation ratio $\frac{R}{I} = N$, Eckenfelder (1961) presented the following formulas

$$\frac{L_e}{L_o} = \frac{1}{(1 + N)(1 + 2.5 \frac{D^{.67}}{Q_1^{.5}}) - N} \tag{27}$$

where L_o is the BOD of undiluted waste and Q_1 is the diluted hydraulic rate in million gallons per acre per day on the filter.

The diluted influent BOD is given by a simple mass balance as

$$L_i = \frac{L_o + NL_e}{1 + N} \tag{28}$$

The application of recirculation to the equation given by Amado (1964), i.e., the combination of equation (28) and

equation (20) results in

$$\frac{L_e}{L_o} = \frac{0.1 + 0.9 \exp \left\{-0.61 \frac{D^{0.628}}{[(1 + N)Q_2]^{0.44}}\right\}}{0.1 + 0.9N \left\{1 - \exp \frac{-0.61D^{0.628}}{[(1 + N)Q_2]^{0.44}}\right\}} \qquad (29)$$

where here Q_2 is the undiluted hydraulic rate in cubic feet per square foot per hour, and the depth D is in feet.

The removability factor which appears as a result of recirculation has not been taken into account in equations (27) and (29).

In practice, recirculation is an established fact with the operation of high rate trickling filters. It contributes to the uniformity and equalization of applied load, and flexibility of plant operation through the adjustment of the rate of recirculation. It is also recognized that recirculation has a beneficial seeding effect on plant influent. The optimum equalization rate of recirculation is dependent on economic considerations and in general, very high rates of recirculation become uneconomical.

NEW CONCEPTS

In general, a continuous flow system consists of some form of say reactor such as activated sludge, trickling filter, sedimentation basin, etc., the purpose of which is to reduce some undesirable property, e.g., BOD, solids, etc., associated with the flow. The degree of success of the operation, i.e., the efficiency of the system, depends mainly on the following two factors (Herbert et al, 1956).

1. The flow pattern, i.e., the way in which material passes through the system, which depends on the physical design of the system.

2. The kinetics of the reaction taking place, i.e., the mechanism by which, for example, BOD is removed.

Flow Pattern

The flow pattern can be characterized by the distribution of residence times of flow particles passing through the system (Danckwerts, 1953). The two extreme cases of flow pattern are plug flow and completely mixed flow. However, in the majority of real cases, the flow type falls in between the two.

1. Completely Mixed Case. The age distribution of the particles in the effluent of this type of flow ranges from zero to infinity. If we denote the percentage of particles having residence times, i.e., ages, between o and t by F, then F is given by

$$F = 1 - e^{-\frac{Q}{V}t} \qquad (30)$$

where V is the volume of the reactor and q is the flow rate. See Figure (1).

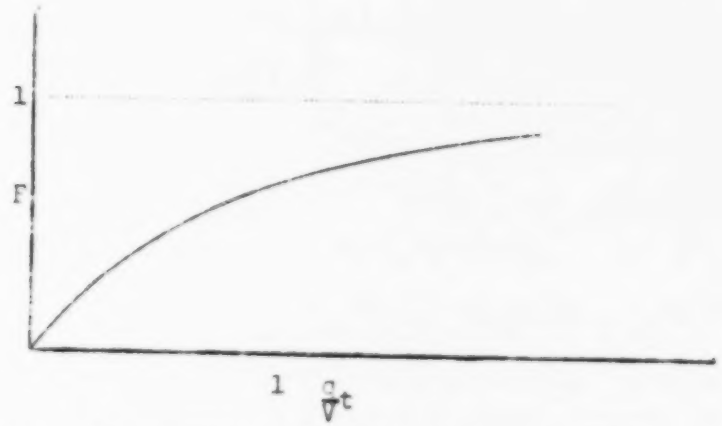

Fig. 1. Flow Pattern for Complete Mixing

2. Plug Flow Case. Here all the fluid particles in the effluent have stayed in the tank an equal amount of time, i.e., they all have the same age which is equal to $\frac{V}{q}$. In this case F is shown in Figure (2).

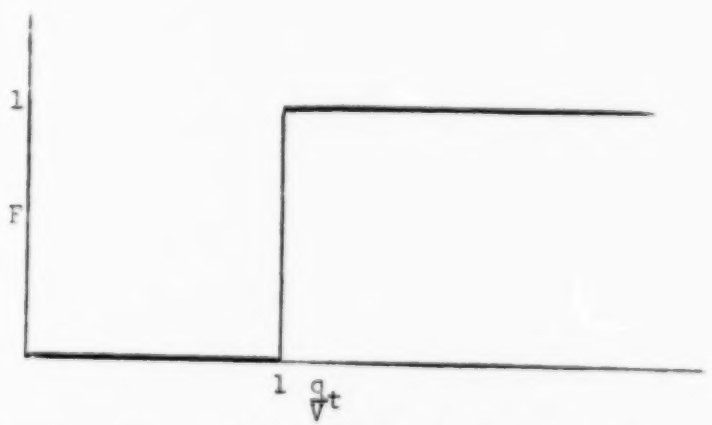

Fig. 2. Flow Pattern for Plug Flow

3. Mixed Flow. In general flow cases there occurs longitudinal mixing and short-circuiting which renders a mathematically undefined flow pattern looking like Figure (3), the shape of which can be determined through tracer techniques.

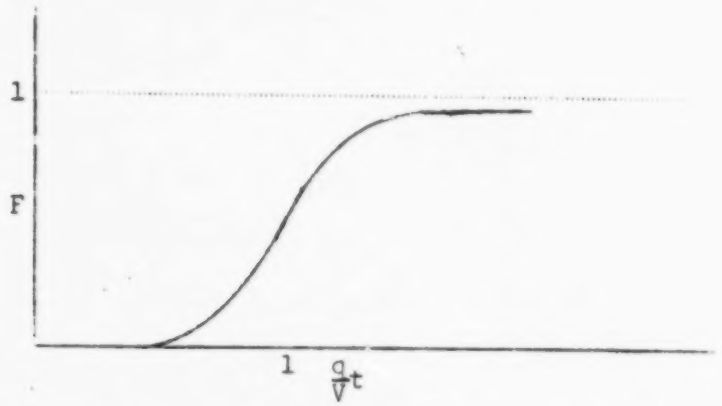

Fig. 3. Flow Pattern for General Case

To demonstrate the importance of the flow pattern, suppose we have a system in which a first order reaction, for example, that of the decay of radioactivity, is occurring. The performance, P, of this system or the fraction of the concentration remaining in the effluent is given by

$$P = \int_{t=0}^{\infty} e^{-kt} \frac{dF}{dt} dt \qquad (31)$$

where K is the first order reaction rate constant. If the flow is completely mixed, then

$$F = (1 - e^{-\frac{Q}{V}t}) \qquad (32)$$

and

$$P = \int_{t=0}^{\infty} e^{-kt} \frac{d}{dt}\left(1 - e^{-\frac{q}{V}t}\right) = \frac{1}{1 + \bar{t}K} \tag{33}$$

where $\bar{t} = \frac{V}{q}$.

In the case of plug flow, we have $\frac{dF}{dt} = 0$ for $t \neq \bar{t}$ and $\frac{dF}{dt} = \infty$ for $t = \bar{t}$ and the performance is given by

$$P = \int_{t=0}^{\infty} e^{-Kt} \frac{dF}{dt}\, dt = e^{-K\bar{t}} \tag{34}$$

These two results, i.e., equations (33) and (34) we are all familiar with. For the case of Figure 3, graphical method is used to determine the efficiency.

Usually the calculations for the performance of a system are based on either plug flow or completely mixed flow and then on the basis of these calculations, some conclusions are drawn concerning the reaction kinetics of the system. These calculations are not exactly correct when dealing with real flow patterns which occur in the system deviating from the assumed ideal case.

The attainment of plug flow in practice is very difficult. However, it is usually assumed that the flow of liquid through a packed column approximates that of a plug flow with the following factors affecting the degree of approximation: the viscosity, density and velocity of the liquid and the size and shape of the packing material.

Another way of approximating this type of flow is by a series of small vessels with complete mixing in each arranged in a long chain (Powell et al, 1962). To attain a true plug flow, we increase the number of vessels without limit while their volume approaches zero in such a way that the total volume of the chain remains constant. We are currently investigating the possibility of trying to simulate the trickling filter with such a series of vessels.

Reaction Kinetics

It is often assumed that the kinetics of biological oxidization of organic material follows a first order reaction. The reasoning is based more on convenience and empirical results than on theory.

A direct consequence of biological utilization of organic material is the formation of new cells which in turn utilize the available nutrient. So it is important that consideration be given not only to BOD concentration, but also to the concentration of bacteria. A simple and practical rate formulation which considers both these factors is that proposed by Keshevan, Behn and Ames (1964), i.e.,

$$\frac{dL}{dt} = -KLN \qquad (35)$$

where N is bacterial concentration. If we assume that the increase in the concentration of bacteria is proportional

to the amount of BOD removed, i.e.,

$$\frac{dN}{dL} = -a \tag{36}$$

then the combination of formulas (35) and (36) results in a logistic type of rate equation used for population forecasting. This fact, and the fact that population dynamics of different species have inherently basic similarities, could be considered as an indirect support for the soundness and applicability of equation (35).

Applications to Trickling Filters. In the field of chemical engineering, it is usually assumed that flow through a packed column could be approximated as plug flow, i.e., the flow pattern corresponds to Figure 2. If we consider that this is true of our trickling filter, and that BOD removal rate occurs by first order reaction, then the correct formula to use would be

$$\frac{L}{L_o} = e^{-kt} \tag{37}$$

This, of course, formed the starting point for analysis such as that by Schulze (1960), Eckenfelder (1961), etc. This is tantamount to assuming, in the case of the analysis by Ames, Behn and Collings (1962) that the resistance to adsorption is very small compared to resistance to reaction, i.e., kA_v is very large compared to $\frac{K}{a}$ in the equation

$$\frac{1}{K_m} = \frac{1}{kA_v} + \frac{a}{K} \tag{18}$$

If we prefer to use an equation with the concentration of bacteria expressed in it, we should use the basic differential form:

$$\frac{dL}{dt} = -KLN \qquad (35)$$

The integration of equation (35) is not a straightforward matter because of the limits one would place on the bacteria from an initial value of N_o to N will not have its physical counterpart.

We could resort to equation (35), writing down by analogy:

$$\frac{1}{K_m} = \frac{1}{kA_v} + \frac{\alpha}{KN} \qquad (38)$$

This leads to another difficulty in that as $\frac{KN}{\alpha}$ increases and approaches the value of kA, they both have to be taken into account. While there has been ample work done on trickling filters, we cannot say with any degree of assurance just what the parameters are in value in equation (38).

While waiting for basic research to improve the understanding of the mechanism of the trickling filter, the design engineer may apply the equation due to Eckenfelder or that due to Amado with confidence that it will lead in the correct direction.

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